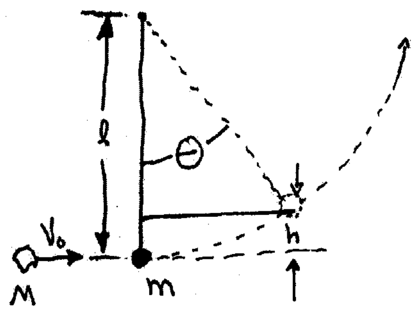
Problems:

(1) A small ball of mass m hangs vertically at the end of a light string. The string has length ℓ. A clay ball with mass M and speed v_o collides and sticks to the ball.

(a) What is the speed of the ball at an angle θ?

(b) What initial speed v_o must the clay ball have to ensure that the ball reaches the top of the circular path without the string becoming slack? Explain your reasoning.



AFTER THE COLLISION, AND BEFORE THE PENDULUM HAS MOVED FAR, WE HAVE

$$MV_o = (m+M)V \qquad (1)$$

BY CONSERVATION OF MOMENTUM. NOW, BY CONSERVATION OF ENERGY

$$E = \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)V'^2 + (m+M)gh \qquad (2)$$

WHERE V' AND h ARE THE SPEED AND HEIGHT AT ANGLE θ. FROM THE DIAGRAM $h = \ell - \ell\cos\theta$ SO (1) AND (2) GIVE

$$\left(\frac{1}{2}\right)\frac{M^2V_o^2}{(m+M)} = \frac{1}{2}(m+M)V'^2 + \cancel{mg}\ (m+M)g\ell(1-\cos\theta)$$

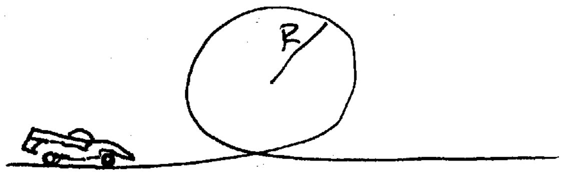
(a) $$\therefore \quad V' = \sqrt{\frac{M^2V_o^2}{(m+M)^2} - 2g\ell(1-\cos\theta)}$$

AT THE TOP, $\theta = \pi$, AND THE MINIMUM SPEED WILL BE OBTAINED WHEN THE TENSION IN THE STRING VANISHES. HENCE

$F = ma = \cancel{(m+M)}g = \cancel{(m+M)}\frac{V'^2}{\ell}$ AND SO $V'^2 = g\ell$. FROM (1) AND (2)

$$\frac{1}{2}\left(\frac{M^2V_o^2}{m+M}\right) = \frac{1}{2}(m+M)g\ell + (m+M)g\ell(2) \qquad \therefore \ V_o \geq \left(\frac{m+M}{M}\right)\sqrt{5g\ell}$$

(2) A spring-powered toy car of mass m sits on a track with a loop of radius R. The spring in the car has a constant k. Find the minimum deflection D that you have to impart to the spring so that the car safely navigates the loop. Assume there is sufficient friction between the cars wheels and the track and that, otherwise, friction is negligible.



INITIALLY

$$E = U_s = \frac{1}{2}kD^2$$

AT TOP OF LOOP

$$E = U_g + K = mg2R + \frac{1}{2}mv^2$$

~~TOP OF~~ TO ENSURE THAT THE CAR COMPLETES THE LOOP

AT TOP $\overset{0}{\cancel{N}} + \cancel{m}g = \frac{\cancel{m}v^2}{R}$ (GIVES MINIMUM v)

HENCE, BY ENERGY CONSERVATION,

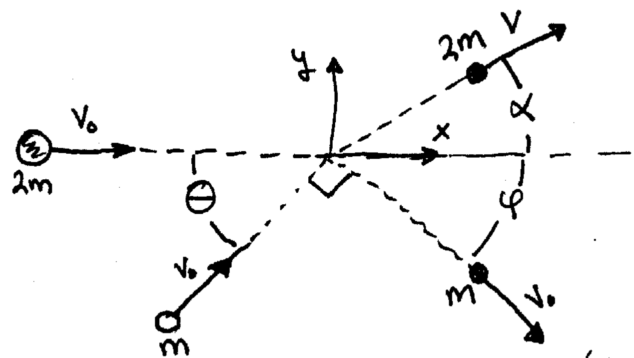
$$\frac{1}{2}kD^2 = mg2R + \frac{1}{2}gRm = \frac{5}{2}mgR$$

$$\Rightarrow D = \sqrt{\frac{5\cancel{2}mgR}{k}}$$

(3) Two objects initially moving at the same speed, 3 m/s, collide at an angle of $\theta = 30^o$. One object is twice as massive as the other. The lightest mass is deflected by $\pi/2$. But its speed is unchanged.

(a) What is the deflection angle, α, of the $2m$ mass?
(b) What is its speed?
(c) Is the collision elastic?

USE CONSERVATION OF MOMENTUM IN COMPONENTS



$\theta + \frac{\pi}{2} + \varphi = \pi$

$\Rightarrow \varphi = \frac{\pi}{2} - \theta$

x: $2\cancel{m}V_0 + \cancel{m}V_0\cos\theta = \cancel{m}V_0\cos\varphi + 2\cancel{m}V\cos\alpha = \cancel{m}V_0\sin\theta + 2\cancel{m}V\cos\alpha$

y: $\cancel{m}V_0\sin\theta = -\cancel{m}V_0\cos\theta + 2\cancel{m}V\sin\alpha$

OR, WITH $\theta = 30^\circ$,

$$2V_0 + \frac{\sqrt{3}}{2}V_0 = \frac{V_0}{2} + V\,2\cos\alpha$$

$$\frac{V_0}{2} = -\frac{\sqrt{3}}{2}V_0 + 2V\sin\alpha$$

DIVIDING GIVES $\Rightarrow \tan\alpha = \frac{1+\sqrt{3}}{3+\sqrt{3}}$

a.) $\therefore\ \alpha = 30^\circ \Rightarrow$ (b.) $V = V_0\left(\frac{1+\sqrt{3}}{2\sin\alpha}\right) \simeq 4.1$ m/s

(c.) NO, SINCE $\Delta K = \frac{1}{2}(2m)(V^2 - V_0^2) < 0$.

(4) An ice skater performs a pirouette (a fast spin) by pulling in his outstretched arms close to his body.
  (a) What happens to his angular momentum about the axis of rotation?
  (b) What happens to his rotational kinetic energy about the axis of rotation?
  (c) What happens to his moment of inertia about the axis of rotation?

(a.) NOTHING, $\vec{L}$ IS CONSERVED

(b.) $L =$ CONSTANT $= I_o \omega_o = I\omega$ SO AS I DECREASES
ω INCREASES. $K = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{L^2}{I}$ $\Rightarrow$ K INCREASES

(c.) I DECREASES AS r DECREASES. ROUGHLY
$I_o = 2mr_o^2$ (FOR HIS ARMS) AND GOES TO
$I = 2mr^2$ WITH $r < r_o$.

(5) A rotating flywheel can be used to store energy. If are designing a flywheel to store 1.00?10^6 J of energy when rotating at 400 rad/s, what is the moment of inertial of the wheel in kg?m^2?

$K = \frac{1}{2} I \omega^2 \quad \Rightarrow \quad I = \frac{2K}{\omega^2} = \frac{2 \cdot 10^6}{(400)^2} \simeq 12.5 \, kgm^2$

$\simeq 10 \, kgm^2$