Physics 350 Fall 2008 Mid-Term exam #1

This is a take-home exam. You may use your normal resources in answering it–your text books, class notes, homeworks and homework solutions etc. You are not to talk to each other about the problems. You should attempt all problems. They are of equal value (10 points per question) (except for some extra credit for problem 8) though not necessarily of equal difficulty. I encourage you to use Maple if it helps. Along lines pioneered by Gordon, I will sell hints at a rate of between 0.5 and 1 point per hint (my judgment) up to a total of 8 points per person. These are in addition to the normal questions of clarification and interpretation that arise in all exams. Because I am out of town for part of the time I strongly encourage you all to contact me by email.

1) Show the following vector calculus equivalences (note that r is the magnitude of $\vec{r}$)

a) $\vec{\nabla} r^n = n r^{(n-2)} \vec{r}$ b) $\vec{\nabla} f(r) = \frac{\vec{r}}{r} \frac{df}{dr}$ c) $\vec{\nabla}(\ln r) = \frac{\vec{r}}{r^2}$ d) $\nabla^2(\ln r) = \frac{1}{r^2}$.

2) A strong softball player hits the ball when it is at a height of 0.7m above home plate. The ball leaves the player's bat at an elevation of 3° and travels toward a fence 2m high and 60m away in center field. Assuming that air resistance is proportional to the square of the ball's speed with a drag coefficient $c_W = 0.5$ and that the 200g softball has a radius of 5cm, how fast must the batter hit the ball for it to just clear the fence?

At this speed what angle of elevation would give the maximum range and, if started at that elevation, by how much would the ball clear the fence?

3) Consider a particle moving in the region $x > 0$ under the influence of the force

$$U(x) = U_0 \left(\frac{a}{x} + \frac{x}{a} \right)$$

where $U_0 = 1J$ and $a = 2m$.

a) Find the force that the particle experiences.

b) Plot the potential, find the equilibrium points, and determine if they are stable or unstable.

c) The particle is released from rest at the point $x = 0.3m$. Discuss its subsequent motion in as much detail as possible. In particular what are the limits of its subsequent motion and what can you say about the frequency of the motion?

4) A gun fires a projectile at some unknown angle θ above the horizontal on a flat, horizontal plane. If the projectile reaches a maximum height h above the plane and lands a distance R from the launch point then show that, for the same initial projectile speed, the maximum range of the gun is given by

$$R_{max} = 2h + \frac{R^2}{8h}$$

5) An oscillator consists of a charged pith ball, mass $m = 10g$, charge $Q = 1\mu C$, and radius $R = 5mm$, suspended in a closed container (so there are no air currents) by a massless string of length $\ell = 0.1m$ so that the ball hangs freely between a large pair of parallel capacitor plates. The plates are driven by a sinusoidal voltage so that there is a uniform electric field round the ball given by E = 1000 cos(ωt).

a) Show that, so long as the motions are small enough to apply the small angle approximation, the ball acts as a driven damped harmonic oscillator.

b) What provides the damping force?

c) What is the Q of the system.

d) Plot the amplitude of the motion as a function of the driving frequency over the range f = 0-3Hz.

e) Over what part of this range is the small angle approximation valid?

You should note that the viscosity of air is $\eta = 0.018 \times 10^{-3}$Pascal seconds.