Physics 195 Spring 2010 Notes for week 7

Basic Facts

Travelling Waves

The basic form of a travelling wave is a function of the form $f(x - vt)$ for a wave travelling from left to right or of the form $g(x + vt)$ for a wave travelling from right to left. In particular the common case of a Gaussian hump with height h and width σ produces a wavefunction of the form

$$y(x,t) = h \times e^{-\dfrac{(x-vt)^2}{2\sigma^2}}$$

while an extended sinusoid has the obvious form

$$y(x,t) = h \times \sin(x - vt + \phi)$$

Any travelling wave on a stretched string can also be written as a sum of normal modes. If the wave is localised to within some region $\Delta L < L$ of the string then the resulting spectrum will be spread over a range of modes Δn. A more localised wave (smaller ΔL) will have a wider spectrum (larger Δn) and vice versa. Indeed, we find in general $\Delta L \times \Delta n \approx$ constant.

Energy and Momentum in Waves

If a wave on a stretched string with tension T and mass per unit length μ has the form y(x,t) then the wave carries energy and momentum along with it as it moves. The energy will be in the form of kinetic energy in the regions where the wave is moving fastest and in the form of potential in those regions where the slope of the wave is greatest.

Useful Facts

Energy and Momentum in Waves

The energy density (energy per unit length) consists of two parts, a kinetic energy part U_T associated with the up-and-down movement of the string and a potential energy part U_V associated with the stretching of the string. They are given by

$$U_T = \frac{1}{2}\mu\left(\frac{\partial y(x,t)}{\partial t}\right)^2 \qquad and \qquad U_V = \frac{1}{2}T\left(\frac{\partial y(x,t)}{\partial x}\right)^2$$

The momentum density $\wp$ is given by

$$\wp = -\frac{1}{2}\frac{\partial y(x,t)}{\partial t}\frac{\partial y(x,t)}{\partial x}.$$