HW 10 Solutions

1) a) $C = \frac{\varepsilon_0 A}{d}$. $A = \pi r^2 = 0.0038\ m^2$. D = 0.025 m. $C = 1.36 \times 10^{-12}$ F.

b) $Q = CV = 3.4 \times 10^{-10}$ coulombs.

c) E = V/d = 250/0.025 = 10,000 V/m.

d) Flux through left side = EA =10,000 V/m × $\pi(0.01m)^2$ = 3.1 $N{\cdot}m^2/C$. E is into cylinder, so by convention, the flux is negative. Flux through the right side is the same but E is out of cylinder so the flux is positive.

e) Flux through sides of cylinder = 0.

f) Flux total = 0 because charge enclosed = 0. There is negative flux through the left side and positive flux through the right side, and no flux through the side of the cylinder, so total flux = 0.

2) a) I = V/R = 7.5/15 = 0.5 A.

b) $B = \mu_0 IN/l$. l = 0.083 m, so B = 0.0036 T.

c) Flux through left face of the cylinder = BA = 0.0036 T × $\pi(0.01m)^2$ = 0.0000011 $T{\cdot}m^2$. The flux through the right face is the same. The flux through one face is positive and the flux through the other face is negative.

d) Flux through the side of the cylinder = 0.

e) According to Maxwell's second equation, the total magnetic flux through a closed surface is always 0. The flux through left face and the flux through the right face cancel because they have opposite signs, and the flux through the side is 0, so the total flux = 0.

3) a) Flux changes from 3.1 $N{\cdot}m^2/C$ to 0 in 0.15 s, so the average rate of change is $\Delta\Phi_E/\Delta t$ = 3.1/0.15 = 20.7 $N{\cdot}m^2/C{\cdot}s$

b) By Maxwell's 3rd equation, $B{\cdot}2\pi r = \mu_0\varepsilon_0\Delta\Phi_E/\Delta t$. Solving for B with r = 0.01 m gives $B = 3.7 \times 10^{-15}$ T.

c) The field is extremely weak, about 1 ten billionth of the magnetic field of the earth. This effect was impossible to observe in Maxwell's time, and is difficult to observe even today.

4) a) Flux changes from 0.0000011 $T \cdot m^2$ to 0 in 0.082 s. The average rate of change of flux is $\Delta\Phi_B/\Delta t = 0.0000134\ T \cdot m^2/s$.

b) $E \cdot 2\pi r = -\Delta\Phi_B/\Delta t = 0.0000134\ T \cdot m^2/s$. Solving for E, with r = 0.01 m gives E = 0.00021 N/C.

5) $v = f\lambda$. Solving for λ gives $(3.00 \times 10^8\ m/s)/(1.85 \times 10^9\ s^{-1}) = 0.162$ m. Putting in f = 1.99×10^9 Hz gives 0.151 m. So the wavelength range is 15.1 to 16.2 cm. This is in the microwave part of the spectrum.

6) a) Intensity = Power/Area. The area is $\pi(0.00055m)^2 = 0.00000095\ m^2$. Thus the intensity = 0.00085/0.00000095 = 895 W/m^2. It is called the average intensity because it is the average over one cycle of a wave. As you will see in the next problem, this is about the same as the intensity of direct sunlight.

b) Average intensity = $(\frac{1}{2})\varepsilon_0 c E_0^2$. Solving for E_0 gives E_0 = 821 N/C.

7) The sun's energy is sent out equally in all directions. The total power output is the intensity times the area of a sphere of radius equal to the earth-sun distance. Area = $4\pi R^2 = 2.8 \times 10^{23}\ m^2$. The total power is about 2.8×10^{26} watts. That is a lot of energy.

8) a) From the previous problem the total power per unit area is 1000 W/m^2. Thus the power per unit area in the visible part of the spectrum is 30% of 1000 = 300 W/m^2.

b) Power output as visible light = 6 watt. The area of a sphere with a radius of 0.35 m is $4\pi R^2 = 1.54\ m^2$. Thus the intensity = 6/1.54 = 3.9 W/m^2. This is about 1.3% of the intensity of the sun.

9) a) $A = 4\pi R^2 = 4\pi(0.03)^2 = 0.011\ m^2$. Average intensity = Power/Area = 1/0.011 = 88 W/m^2. Average intensity = $(\frac{1}{2})\varepsilon_0 c E_0^2$. Solving for E_0 gives E_0 = 257 N/C.

b) Since the energy in an electromagnetic wave is shared equally between electric and magnetic fields, we have $\varepsilon_0 E_0^2 = B_0^2/\mu_0$. Thus $B_0 = (\varepsilon_0\mu_0)^{0.5}E_0 = E_0/c = 8.6 \times 10^{-7}$ T.

10) If you are 2.5 m from the mirror, then the image appears 2.5 m behind the mirror and thus it is 5.0 m from you. This is the distance for which the camera should be focused.

11) First figure out the angle of reflection of the ray leaving the floor and going into the person's eye. Make a triangle from the bottom of the mirror, to the person's eye, to a point on the person 43 cm above the floor. From this right triangle use $\tan\theta = (1.68-0.43)/2.20$ m. This gives an angle of 29.6º. This is the angle of reflection of the red ray shown, so it is also the angle of incidence. But then by geometry it is also the angle between the floor and the light ray shown going from the floor to the bottom of the mirror. The tangent of this angle is 43/x. Solving for x gives x = 43 cm/tan(29.6º) = 76 cm.

12) The rays in red are from the left ear and the rays in blue are from the right ear. I have shown 4 rays from each ear. For example, for the left ear there is a ray that hits M1 at normal incidence, there is a ray that hits M2 at normal incidence, and there is a ray that first hits M1 and then hits M2 and there is a ray that first hits M2 and then M1. The doubly reflected rays are the ones that produce the L12 image. The right ear is similar.