HW 12 Solutions

1) Ch23 P31

The angle between the light beam and the surface of the pool is given by $\tan\theta = 1.3/2.7$. This gives $\theta = 25.7^\circ$. The angle of incidence is $90^\circ - 25.7^\circ = 64.3^\circ$. Using Snell's law, the angle of refraction is given by $\theta_r = \sin^{-1}\left(\frac{1}{1.33}\sin(64.3)\right) = 42.6^\circ$. This light travels 2.1 m vertically before hitting the bottom. The horizontal distance it travels, x, is given by $\tan\theta_r = x/2.1$, which gives x = 1.93 m. Measure from the wall, the light hits the bottom of the pool at 1.93 m + 2.7 m = 4.63 m or 4.6 m.

2) Ch23 P73

This is very similar to the previous problem, but the light ray is now traveling in the opposite direction. The light ray in air make an angle relative to the normal of $90^\circ - 14^\circ = 76^\circ$. Using Snell's Law, the light traveling in the water must have hit the water surface at an angle of incidence given by $\theta_i = \sin^{-1}\left(\frac{1}{1.33}\sin(76)\right) = 46.8^\circ$. Since the pool has a width of 5.50 m, the depth of the pool, d, is given by $\tan\theta_I = 5.50/d$. Solving gives d = 5.16 m or 5.2 m.

3) Ch24 P16

From the index of refraction graph the index at 450 nm appears to be about 1.65 and the index at 650 nm appears to be about 1.62. Your numbers may be slightly different.

The light enters the prism at a 45° angle from the normal. Thus the angle of refraction is $\theta_r = \sin^{-1}\left(\frac{1}{1.65}\sin(45)\right) = 25.4^\circ$ for the short wavelength and $\theta_r = \sin^{-1}\left(\frac{1}{1.62}\sin(45)\right) = 25.9^\circ$ for the long wavelength. For the short wavelength, the light in the prism makes an angle of 90 – 25.4 = 64.6° from the air/glass surface. Since the apex angle is 60°, the ray hits the other side at 180 – 60 – 64.6 = 55.4° relative to the surface, or 34.6° relative to the normal. Applying Snell's Law again gives $\theta_2 = \sin^{-1}\left(\frac{1.65}{1}\sin(34.6)\right) = 69.5^\circ$. Working through the geometry and math in the same way for the longer wavelength gives $\theta_1 = \sin^{-1}\left(\frac{1.62}{1}\sin(34.1)\right) = 65.2^\circ$. Note that the red light (650 nm) is deviated less than the violet light.

4)

a) The colors are reversed. Red light should bend least and violet most, so red should be on top and violet on the bottom in the view on the right.

b) The different colors are coming together after going through the prism instead of spreading out.
c) The red light is bending up as it leaves the glass instead of bending down. This is related to b).
d) As it enters the prism the violet light hardly bends at all, then as it leaves the prism it bends a lot even though its angle of incidence is close to 0º.

5)
a) For TIR to occur the light must hit the surface at greater than the critical angle. In this case the light hits the surface at 45º, so set this as the critical angle. The critical angle is the angle at which the light would leave at a 90º angle so $n_{glass}\sin(\theta_{crit}) = (1\sin(90))$.
With $\theta_{crit} = 45°$ this gives his gives $n_{glass} = \dfrac{1}{\sin 45°} = 1.414$.

b) First find the angle of refraction as the light enters the glass: $1\cdot\sin 5° = 1.414\cdot\sin\theta_r$. Solving gives $\theta_r = 3.53°$. Thus the ray in the glass is traveling at 90 – 3.53 = 86.47º from the surface. Using geometry, the light hits the diagonal surface at an angle of 180 – 86.47 – 45 = 48.5º from the diagonal surface. This means the angle of incidence is 90 – 48.5 = 41.5º. Since the critical angle is 45º if n_{glass} is 1.414, the light hits at less than the critical angle and will not undergo total internal reflection. If will still undergo partial reflection of course. To make sure it undergoes total internal reflection, the index of refraction would have to be greater than 1.414.

6) Ch23 P41
The light enters the fiber at an angle α. If $\alpha = 0$, then $\beta = 0$ and the light travels parallel to the fiber and will never hit the top surface. As α increases, so does β, which means γ decreases, because $\gamma = 90 - \beta$. Remember that TIR occurs if the angle of incidence is *greater* than the critical angle. As α is increased, if γ gets too small, it will be less than the critical angle. We need to guarantee that does not happen. Clearly the worst case is if $\alpha = 90°$. In that case β will be a maximum and γ will be a minimum. Assume that $\alpha = 90°$ and apply Snell's Law to the first surface. We get $1\cdot\sin 90° = n_{glass}\cdot\sin\beta$. Solving for β gives $\beta = \sin^{-1}\left(\dfrac{1}{n_{glass}}\right)$. Substituting into $\gamma = 90 - \beta$ gives

$$\gamma = 90 - \sin^{-1}\left(\frac{1}{n_{glass}}\right).$$

We need γ to be greater than or equal to θ_{crit}. But we know that for light traveling in glass and hitting a glass-air interface, $\theta_{crit} = \sin^{-1}(1/n_{glass})$. So we have

$$90 - \sin^{-1}\left(\frac{1}{n_{glass}}\right) \geq \sin^{-1}\left(\frac{1}{n_{glass}}\right).$$

This equation will be true if $\sin^{-1}\left(\frac{1}{n_{glass}}\right) \le 45°$. Taking the sine of both sides gives $\left(\frac{1}{n_{glass}}\right) \le \sin 45 \approx 0.707$, which means $n_{glass} \ge 1.414$.

Alternatively, you can do the problem numerically and show that n=1.414 works:

n: 1.414 θ crit 45.0

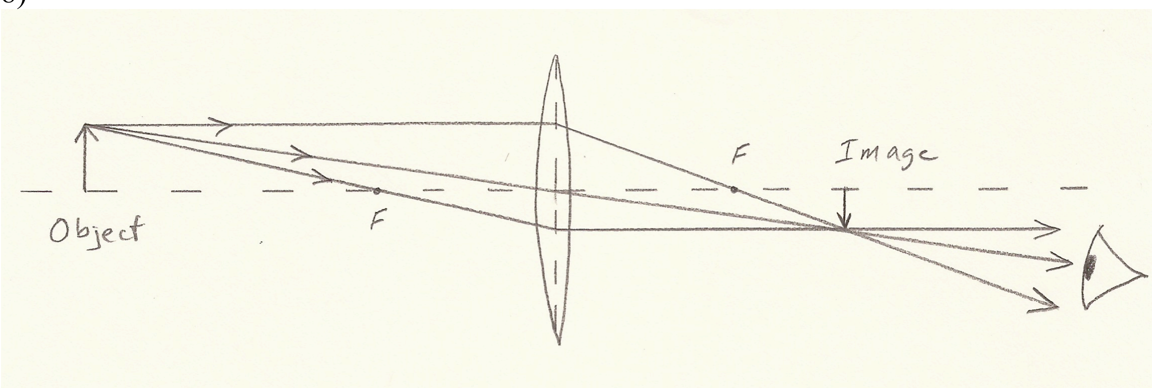
alpha	beta	gamma
10	7.1	82.9
20	14.0	76.0
30	20.7	69.3
40	27.0	63.0
50	32.8	57.2
60	37.8	52.2
70	41.6	48.4
80	44.1	45.9
90	45.0	45.0

Notice that as alpha is increased, gamma gets smaller and thus is getting closer to the critical angle. With n = 1.414, alpha = 90º gives gamma equal to the critical angle, so any n greater than 1.414 will assure that gamma is always greater than the critical angle.

7) Ch23 P44

It is a little unclear what it means by behind the lens, but I will take it to mean on the opposite side of the sun. Since the sunlight is being focused, it is a converging lens, so its focal length is +18.5 cm. Its power in diopters is 1/0.185 = +5.41 D

8)



The image is real. The light entering the eye really is coming from the location of the image. The image is inverted and reduced in size. Using the lens equation with $d_o = 40$ cm and $f = 15$ cm, gives $d_i = 24$ cm and thus $m = - 24/40 = -0.6$. The diagram looks pretty consistent with these values.

9) Ch25 P6

The focal length is 200 mm and it can be made to be 200 mm from the film, so in this case it will be focused for infinity. If the lens is moved to 206 mm from the film, then $d_i = 206$ mm. Using the lens equation gives $\frac{1}{d_o} + \frac{1}{206} = \frac{1}{200}$. Solving gives $d_o = 6867$ mm $=6.867$ m. The lens can be focused for any distance between 6.9 m and infinity.

10) Ch25 P12

This person is farsighted. The near point is 115 cm from the person's eye, so it is 113 cm from the lens used to correct the vision. We want the person to be able to see an object that is 55 cm from the eye, or 53 cm from the lens. Thus we want a focal length so that $d_i = -113$ cm when $d_o = 53$ cm. The minus sign is there because the image must form on the object side of the lens. Solving $\frac{1}{53} + \frac{1}{-113} = \frac{1}{f}$ gives $f = 100$ cm $= 1.0$ m. The power is $1/f = 1/1.0 = +1.0$ diopters. If we ignored the 2 cm eye to lens distance we would get pretty much the same answer ($\frac{1}{55} + \frac{1}{-115} = \frac{1}{f}$, $f = 105$ cm, power $=+0.95$ diopters). Usually the eye-to-lens distance is ignored because the small effect it introduces is normally not significant. +1 diopter is the power of the weakest reading glasses sold at drug stores.

11) Ch 25 P13

This person is nearsighted. The far point is 14 cm. We want a lens so that an object at infinity will form an image 14 cm from the eye. 14 cm from the eye is 12 cm from the glasses. The image will form on the object side of the lens so the image distance is -12 The lens equation gives $\frac{1}{\infty} + \frac{1}{-12} = \frac{1}{f}$. Thus $f = -12$ cm $= -0.12$ m. The power is $1/-0.12 = -8.3$ diopters.

With contact lens we use $d_i = -14$ cm. The result is $f = -0.14$ m and power $= -7.1$ D

12)

a) $\tan\theta = 0.15/20$. This gives $\theta = 0.43^\circ$ or 0.00749986 radians. Notice that $.15/20 = .0075$, so $\theta \approx \tan\theta$ is a very good approximation.

b) The object distance is $d_o = 4.5$ cm and the focal length is $f = 5.0$ cm. Using the lens equation gives $d_i = -45$ cm. She will be able to focus on this image because it is further than her near point.

c) Using $m = -d_i/d_o$ gives $m = -(-45)/4.5 = +10$. Thus the length of the image of the beetle is $10 \cdot 0.15 = 1.5$ cm.

d) $\tan\theta = 1.5/45$, so $\theta = 1.9^{\circ}$ or .03332 radians. Using the small angle approx you would get $\theta \approx 1.5/45 = 0.03333$ radians, so again the approximation is excellent. The angular magnification is $0.033/0.0075 = 4.4$