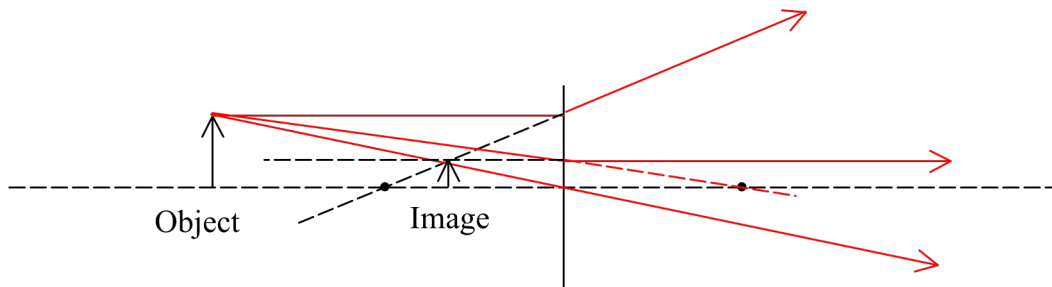


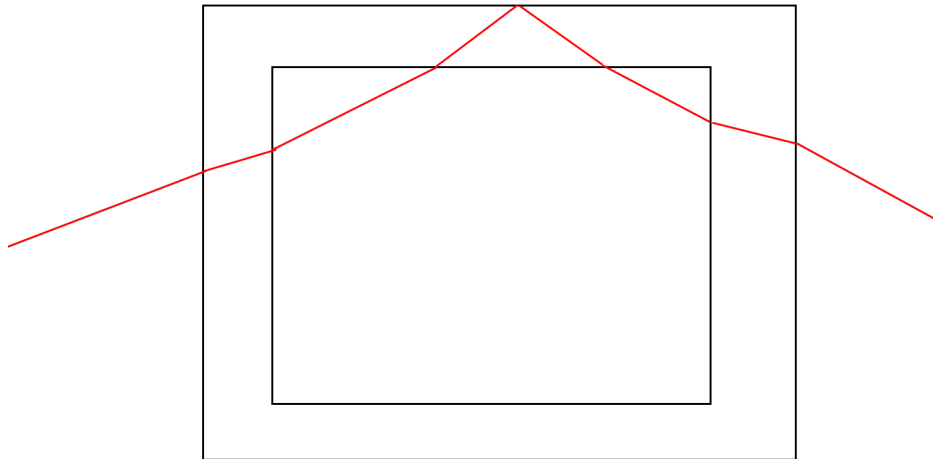
## HW 13 Solutions

1) The mirror forms an image of the camera. Using the mirror equation with  $f = R/2 = 40$  cm and  $d_o = 120$  cm gives an image distance of 60 cm. Thus the mirror forms an image of the camera that is 60 cm from the mirror. This image is then  $120 - 60 = 60$  cm from the camera. Using the lens equation for the camera with  $f = 50$  mm = 5.0 cm and  $d_o = 60$  cm gives an image distance of 5.45 cm. Thus, the distance from the lens to the film is 5.45 cm.

2)



3)



The ray bends toward the normal at the air/glass interface. The angle of refraction, found using Snell's Law, is  $22.2^\circ$ . It hits the glass/water interface at an angle of incidence of  $22.2^\circ$ . It bends away from the normal with an angle of refraction of  $25.6^\circ$ . It travels through the water and hits the water/glass interface at an angle of incidence of  $90 - 25.6 = 64.4^\circ$ . It enters the glass and bends toward the normal with an angle of refraction of

52.1°. It hits the glass/air interface at an angle of 52.1° measured from the normal. This is greater than the critical angle, which is  $\sin^{-1}(1/1.52) = 41^\circ$ , so the ray reflects and hits the glass/water interface at an angle of 52.1°. It then repeats the angles. The angle of refraction in the water is 64.4°. The angle of incidence at the water/glass interface is 25.6°. The angle of refraction is 22.2° at the glass/air interface, so it leaves the glass at an angle of refraction of 35°, the same angle at which it entered on the other side. Symmetry is nice!

4)  $d\sin\theta = m\lambda$ .  $d = 0.016 \text{ mm}$ ,  $\theta = 8.8^\circ$ ,  $m = 5$ .  $\lambda = 0.000490 \text{ mm} = 490 \text{ nm}$ .

5)  $d\sin\theta = m\lambda$ .  $\sin\theta \approx \tan\theta = 38\text{mm}/2000\text{mm}$ .  $\lambda = 680 \text{ nm}$ .  $m = 4$ .  $d = 0.00014 \text{ m}$

6) Numbers will vary considerably, but here are some typical values. The distance from the central maximum to the first order maximum is about 1.0 cm.  $\tan\theta = 1/5 = 0.2$ . This is a pretty small angle so you could use that  $\sin\theta \approx \tan\theta$ . However, you could also find  $\theta = \tan^{-1}(0.2) = 11.3^\circ$ . Taking the sine gives  $\sin\theta = 0.196$ , showing that the small angle approx. works quite well. Since  $d$  was set at 1.0 cm and  $m = 1$ , we get  $\lambda = 1.0 \cdot 0.196 = 0.196 \text{ cm}$ .

Measuring 10 wavelengths on just one of the sheets, not on the interference pattern formed by two sheets, I got about 18 mm, so  $\lambda = 0.18 \text{ cm}$ .

It is better to measure 10 wavelengths because it reduces the uncertainty in a single wavelength.

7)

a)  $d_o = 6 \text{ cm}$  and  $f = 10 \text{ cm}$ . This gives  $d_i = -15 \text{ cm}$ , so the image forms on the same side of the lens as the object. The magnification is  $-(-15/6) = +2.5$ , so the image is upright and magnified.

b) The near point in the problem was stated as 25 cm, but I sent out an email changing it to 12 cm. The angle made by an object placed at the near point is  $\theta$ .  $\tan\theta = h_o/12 \approx \theta$ . The angle when viewed through the magnifier is given by  $\tan\theta' = h_i/d_i$ , except that we do not worry about the fact that  $d_i$  is negative. For the  $\tan\theta'$ , we only care about the magnitude of  $d_i$ . So  $d_i = 15$  and  $h_i = 2.5 h_o$ , from part a. The angular magnification is  $\theta'/\theta = (2.5h_o/15)/(h_o/12) = 2.5 \cdot 12/15 = 2.0 = m_{\text{angular}}$ .

c) When the object is at 10 cm, the image is at  $-\infty$  and the angular mag is  $N/f = 12/10 = 1.2$ .

d) When the object is placed so that the image forms at the near point, the angular mag is  $N/f + 1 = 2.2$ .

Note that c) and d) give the upper and lower limits for the magnification available with this focal length lens for a person with a near point of 12 cm. The magnification for the actual placement of the object used in the lab is somewhere between the upper and lower limits.

8) The problem says assume a relaxed normal eye. That means an eye with a near point of 25 cm that is focused for  $\infty$ .

a) If the eye is focused for infinity, then the image formed by the eyepiece is at infinity. That implies that the image formed by the objective is located at the focal point of the eyepiece. The distance between the lenses is 16 cm but the image formed by the objective forms at the focal point of the eyepiece, which is 1.8 cm from the eyepiece. That means the image formed by the objective has  $d_i = 16 - 1.8 = 14.2$  cm. Using the lens equation for the objective with a focal length of 0.8 cm gives  $d_o = 0.85$  cm.

b) The magnification of the objective is  $-d_i/d_o = -14.2/0.85 = -16.7$ . The magnification of the eyepiece is  $25/1.8 = 13.9$ . The resultant magnification is  $-16.7 \cdot 13.9 = -232$ .

Normally we don't care about the minus sign and the mag is just **232x**

Notice that we have combined linear and angular magnifications. 232x is how big the image would be on our retina with the microscope compared to how big it would be naked eye. The objective produces a larger image of the object, which we could view directly at our near point. Then it would appear 16.7 times bigger than the object and create an image on our retina that was 16.7 times bigger than if we positioned the object at our near point. The eyepiece then produces an angular magnification of 13.8.

9) a) The image forms behind the retina, and no matter how much the physicist strains he can not bend is lens enough to make the image form on his retina.

b) When wearing contacts he is just a "normal" far-sighted person with a near point of 3 m = 300 cm. He wants a near point of 25 cm. In other words he wants to be able to see something that is placed 25 cm from his eye or 23 cm from his glasses. This is the object distance. In order to see it, the lens must produce an image at 300 cm, on the same side of the lens as the object. Thus  $d_i = -300$  cm, or actually -298 cm. **Solving for f gives f = 24.9 cm or 0.249 m. Thus the power in diopters is about 4.0 D.**

10) The first order minimum is given by  $d \sin \theta = \lambda$ . The  $\sin \theta$  is approx  $\tan \theta = x/L$ , so  $dx/L = \lambda$ .  $d = 1.0$  mm,  $L = 5000$  mm, and  $\lambda = .000450$  mm. Solving for x gives  $x = 2.25$  mm. The central maximum is twice this wide = 4.5 mm.

11) You can do this problem without knowing the wavelength, or the distance to the screen, but you can use those pieces of information if you want. The angle of the first order diffraction minimum is given by  $D \sin \theta = \lambda$ . The angle of the fourth order

interference maximum is given by  $d\sin\theta = 4\lambda$ . Since the angles are the same we get  $D = d/4 = 0.063 \text{ mm} = \text{the width of the slit}$ .

12) The distance between the slits is  $1/6100 = 1.64 \times 10^{-4} \text{ cm}$ . Using  $d\sin\theta = m\lambda$ , we find

$$\theta_{1\text{violet}} = 14.8^\circ$$

$$\theta_{2\text{violet}} = 30.8^\circ$$

$$\theta_{3\text{violet}} = 50.2^\circ$$

$$\theta_{1\text{red}} = 23.4^\circ$$

$$\theta_{2\text{red}} = 52.5^\circ$$

$$\theta_{3\text{red}} = \text{does not exist}$$

The third order red does not exist because the formula gives a  $\sin\theta$  greater than 1. This is because the slits are so close together that there is no way to get a difference in distance traveled of 3 wavelengths.

The third order violet is at a smaller angle than the second order red. In other words the colors are not quite in order. Starting from the central maximum and looking at bigger and bigger angles you would see violet in first order than red in first order, then violet in second order, but then before you see the red in second order you see the violet in third order.