HW #9 Solutions

1) a) $F_T = mg = .00444 \text{ kg} \times 9.80 = 0.0435 \text{ N}$

b) Since the tension decreases, the force on the sphere due to the electric field must be up. Since the sphere has a negative charge, the field must be down. Thus the top plate must be connected to the positive terminal of the battery.

c) The sum of the forces on the sphere is 0. The tension exerts an upward force, as does the electric field. Gravity is down. Thus $mg = F_T + F_E = F_T + qE$. This equation can be solved for q. $q = (mg - F_T)/E$. E is given by $E = \Delta V/d = 45.4 \text{ V}/.0327 \text{ m} = 1388 \text{ V/m}$.
Then $q = (0.0435-0.0418)/1388 = 1.22 \times 10^{-6}$ C.

2) a) The magnetic field of the solenoid points down through the solenoid and through the metal ring. Since the field was initially zero inside the ring, the induced current must produce a magnetic field up to counter the change in flux. To produce an upward field, the induced current must be counterclockwise as viewed from above.

b) The magnetic field of the solenoid is down, but it is not uniform, it spreads out like any dipole field. Thus the field of the solenoid has a component in, toward the axis of the solenoid. This was discussed in class on Monday. The inward component of the magnetic field creates an upward force on the current induced in the ring as shown by the right hand rule. The induced current is counterclockwise, the field is horizontal and directed toward the axis, so the force is up.

c) If the current in the solenoid is reversed, the induced current in the ring is reversed too. But the field produced by the solenoid is also reversed, so it now points out, away from the axis of the solenoid. The clockwise current interacting with the horizontal field of the solenoid, pointing away from the axis, creates an upward force.

3) The falling magnet induces eddy currents in the tube. These currents produce an upward force on the magnet slowing its descent. The currents quickly die out after the magnet passes by because of the resistance of the metal tube, but in the process they produced heat energy. Thus the gravitational potential energy was converted into mostly heat energy and a little into KE. The "missing" energy is all heat.

4) a) The switch is open so current = 0. Voltage drop across the internal resistance = 0, so $V_{AB} = 16$V.

b) No current so $V_{CD} = 0$.

c) First find the effective resistance of the entire circuit. The parallel combination gives a resistance of 4 Ω. This is in series with the 3 Ω and the 1 Ω, giving a total resistance of 8 Ω. The current is $I = V/R = 16/8 = 2$ A. This is the total current leaving the battery, which is the same as the current through the 3 Ω resistor.

d) Now there is a voltage drop across the internal resistance. It is $V = IR = 2 \times 1 = 2$ V. Thus the potential difference between A and B is 16 - 2 = 14 V. This is the terminal voltage or output voltage of the battery when the current is 2 A.

e) The current splits when it gets to point D. More current will go through the 6 Ω resistor. The volt drop across the 3 Ω resistor is V = IR = 2 × 3 = 6 V. Thus the voltage across the 6 Ω resistor is 8 V. Using I = V/R one finds I = 8 V/6 Ω = 1.33 A.

f) The potential drop across the 12 Ω resistor is 8 V, the same as across the 6 Ω resistor.

g) Power dissipated = $VI = I^2R = V^2/R = 6 \times 2 = 2^2 \times 3 = 6^2/3 = 12$ watts.

5) There are only two choices, so make a guess that the lower pole of the magnet is a North pole. This creates a field through the coil that is downward, and since the magnet is moving down, the field is getting stronger. To counter this change in flux, the current induced in the coil must produce a magnetic field that is up. A counterclockwise current does indeed produce a magnetic field that is up, so I guessed correctly. The bottom pole of the magnet is a North pole.

6) First figure out the acceleration of the bar. It accelerates from 0 to 28 m/s so its average speed is 14 m/s. But average speed is Δx/Δt, and Δx = 1m. From this we get $\Delta t = \Delta x/v_{ave} =$ 0.0714 s. Acceleration = a = Δv/Δt = 28/0.0714 = 392 m/s^2. You could also get this from $v_f^2 = v_0^2 + 2a\Delta x$.

From Newton, F = ma = 0.0015 kg × 392 m/s^2 = 0.588 N.

The force on the bar is F = I*l*B. Solving for I gives I = F/*l*B = 0.588/(0.22× 1.7 T) = 1.57 A.

7) The electron is moving in a circle at a uniform speed so the time to make one orbit is found from v = d/t, where d = 2πr. Thus to find the time we need the speed and the radius. For an electron gun, conservation of energy can be used to find the speed. qV =(½)mv^2. Solving gives $v = (2qV/m)^{0.5} = 1.07 \times 10^7$ m/s.

The radius can be found using $mv^2/r = qvB$. Thus r = mv/qB = 0.053 m, and t = d/v = 2πr/t = 3.1×10^{-8} s.

Thanks to Eric Maertin for pointing out that the time is actually independent of the voltage. In other words, you did not need to know the voltage to do this problem. This is because the faster the electron goes, the bigger the circle, so it still takes the same amount of time to go around once. Mathematically, you can just say that t = 2πr/v = 2π(mv/qB)/v = 2πm/qB = 3.1×10^{-8} s.

8) a) By the right hand rule for a long straight current, the direction of the magnetic field is east. It is also horizontal.

b) We are only interested in the horizontal component of the earth's field because that is what determines the compass needle direction. The field due to the earth is north and the field due to the wire east. Using a vector diagram for the two fields one can show that in order for the resultant field to be at a 14º angle from north, the tan(14º) = B_{wire}/B_{earth}. Solving for B_{wire} gives $B_{wire} = \tan(14^o) \times B_{earth} = 1 \times 10^{-5}$ T.

c) The field of a long straight wire is given by $B = \mu_0 I/2\pi r$. Solving for I gives $I = B(2\pi r)/\ \mu_0 = 1 \times 10^{-5}\ (2\pi(0.05))/4\pi \times 10^{-7} = 2.5$ A

9) The force required to pull the loop at a constant velocity is just equal to the force on the induced current due to the magnetic field, F = I*l*B. B = 0.550 T and *l* = 0.35 m, but we need to know the current, I.

The current is given by emf/R and we know R = 0.23 Ω so we need to find the emf.

The emf is just the rate of change of magnetic flux. Magnetic flux = BA. B is not changing, but A is. The area of the loop in the field is height × width. The height is constant. It is just what was earlier called l = 0.35 m. The width is changing. We are given the velocity, but this is just Δwidth/Δt = 3.4 m/s. So the area is changing at a rate of 0.35 m × 3.4 m/s = 1.19 m^2/s. Thus the flux is changing at a rate of 0.550 T × 1.19 m^2/s = 0.655 T·m^2/s = 0.655 V. This is the emf.

Substituting gives the current = 0.655 V/0.23 Ω = 2.85 A. Substituting gives the force is F = 2.85 A × 0.35 m × 0.550 T = .55 N. This is the force required to pull the loop at a constant velocity.

10) a) R =ρl/A. l = 263,000 m and A = $\pi(0.02)^2$. This gives R = 5.5 Ω, incredibly small resistance for a wire that long.

b) V = IR = 1800 × 5.5 = 10,000 V

c) The voltage at the end of the line is just the voltage at the beginning minus the voltage drop in the line. V at end = 330,000 - 10,000 = 320,000 V.

d) P = VI = 320,000 × 1800 = 576,000,000 W = 576 MW. This is about the amount of power produced by 1 of the 6 reactors at a typical nuclear power plant. It is enough power to meet the needs a city of about 360,000 people.

e) The wasted heat energy per second is the power dissipated in the line = I^2R =1800^2 × 5.5 =17,800,000 watts, or 17.8 MW.

f) 17.8/576 ×100 = 3%. So the efficiency of the line is about 97%.

g) 3 cables will be needed.

h) Each cable is carrying 1800 A and has a resistance of 5.5 ohms, so the total heat energy produced in the 3 cables is 3 × 17.8 MW = 53.4 MW.

Actually the three cables will not transmit quite as much energy as the single cable because if there is 110,000 volts at the beginning, there will be 100,000 at the end of the line, so the total power delivered is 10,000 × 1800 x 3 = 540 MW. The energy lost as heat will be about 10% of the total energy.

The current standard for the largest transmission lines is 768,000 V, giving even better efficiency than the 330,000 V assumed in the problem.